Minimum-Time Turns Using Vectored Thrust

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A steepest-ascent optimization technique is used to determine the optimal controls and trajectories that minimize the time to turn for a high-performance aircraft with thrust-vectoring capability. No constraints were placed on the angles through which the thrust was vectored in order to determine how much range of thrust vectoring would be exploited if it were available. The determined controls and trajectories are then compared against other methods of turning in minimum time to determine the effects and advantages of thrust vectoring. The results indicate that the use of vectored thrust to supplement the aircraft's lift by directing the thrust into the turn can substantially reduce turning times and increase in-flight maneuverability. The greater the velocity at which the turn is initiated, the larger the range of thrust-vectoring capability is used and the greater the reduction in turning time.

Nomenclature

C_{D_0} = parasite drag coefficient
C_{L_α} = lift-curve slope
D = drag
E = specific energy
g = gravitational acceleration
h = altitude
K_1 = induced-drag parameter
L = lift
S_W = wing area
T = thrust
V = velocity
V_C = corner velocity
V_i = initial velocity
W = weight
X = distance (x direction)
Y = distance (y direction)
α = angle of attack
γ = flight-path angle
ϵ = thrust angle of attack
μ = bank angle
ν = thrust sideslip angle
π = throttle control variable
ρ = density
χ = heading angle
$\dot{a}$ = da/dt (a arbitrary)

Introduction

IN air-to-air combat, minimum-time turns are important for both the attacker and the evader. The normal procedure is to bank the aircraft and to use only a component of the lift vector for turning the aircraft, while accelerating or decelerating using the throttle control. The maximum turn rate that an aircraft can obtain is limited by physical constraints; in particular, structural and angle-of-attack limits. Since aircraft designed for similar missions (i.e., fighter aircraft) tend to have similar design considerations and constraints, they consequently exhibit similar turning rates and times.

In an effort to reduce turning time and increase in-flight maneuverability, the U.S. Air Force is currently investigating the use of vectored engine thrust and expects "performance may very well be boosted."[1] By vectoring thrust, it is possible that an aircraft will acquire its corner velocity in less time—the corner velocity being the velocity at which the aircraft achieves its maximum attainable turn rate. Notably, a two-dimensional convergent/divergent vectoring nozzle has been ground-demonstrated successfully.[2] The F-15 advanced technology short-takeoff-and-landing (STOL) demonstrator aircraft with thrust-directing two-dimensional engine nozzles will be flight tested in 1989.[3] The "technology developed in the STOL demonstrator program will 'have far-reaching implications for the Air Force, especially for the next generation of fighters'." This flight demonstration is also considered "...critical for future aircraft programs."[3]

Several studies present optimal controls and trajectories to minimize turning time. Humphreys et al.[4] used a sequential gradient-restoration algorithm to determine the optimal controls required for an aircraft to make a minimum-time turn in three dimensions. For two different sets of initial conditions, a variety of final conditions and thrust-to-weight ratios were considered. Three control parameter variations were used: angle of attack, bank angle, and thrust.

Johnson[5] used a suboptimal numerical technique to find optimal control schedules that minimized the turning time. The same aircraft model and two cases reported in Ref. 4 were used to verify the technique and to compare the effects of in-flight thrust reversing on reducing the time to turn. Finnerty[6] used this same numerical technique and thrust-reversing aircraft, but restricted the maneuver to the vertical plane.

Well and Berger[7] used a different optimization technique, a multiple-shooting algorithm, to investigate minimum-time 180-deg turns. However, since the specific aircraft characteristics they used are different from those given in Ref. 4 and subsequently used by Johnson[5] and Finnerty,[6] a direct comparison of results is impossible. The conclusions of Ref. 7 do serve as a good qualitative check of optimal minimum-time maneuver sequences.

Most recently, Brinson[8] used the aircraft model from Ref. 4 and the suboptimal numerical technique developed by Johnson,[5] but modified both to include sideforce. Optimal control schedules, trajectories, and turning times were reported for three sets of initial conditions, two of which are the same as those used by Johnson[5] and Finnerty.[6] Since the original aircraft model from Ref. 4 was considered unrealistic due to its low induced drag and high thrust-to-weight ratio, Brinson[8] also varied these two parameters while excluding sideforce in an attempt to match the results of Well and Berger.[7] Although

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only moderately successful in this attempt, the results are useful for evaluating the benefits of thrust vectoring to reduce turning time.

The variety of aircraft characteristics and their resulting optimal controls and trajectories and minimum turning times are too numerous to discuss in detail in this paper. Here we shall present only sufficient comparisons to prove the benefits of thrust vectoring as compared to other methods of reducing turning time. More detailed comparisons and results are presented in Refs. 4–9.

Minimum-Time-to-Turn Problem

The problem will be defined in terms of the assumptions made, the maneuver to be flown, the equations describing the motion of the aircraft, the characteristics that model the aircraft, and practical physical constraints on the control variables.

Assumptions

The aircraft is modeled as a point mass over a flat, nonrotating Earth. Atmospheric properties are taken from the NASA 1962 Standard Atmosphere[10] and the gravitational acceleration is assumed to be constant over the small altitude range covered during the maneuver. The lift coefficient is taken to be a linear function of the angle of attack up to the stall limit, while the drag coefficient is a function of the square of the lift coefficient (parabolic drag polar). Since the duration of the maneuver is small, typically on the order of 10 s, the fuel consumed during the maneuver is negligible and the aircraft weight remains constant. The aircraft engine is considered an ideal jet. Since the maneuver covers a small altitude range, the corresponding small variations in atmospheric density have little effect on the engine performance, and the maximum available thrust remains constant. Additionally, it is assumed that the aircraft flies a coordinated turn (i.e., zero sideslip).

The controls are allowed to vary instantaneously. This eliminates the need to consider controller response that, although obviously an important consideration in the implementation of any control schedule, would only cloud the comparison of results against those of studies investigating other ways to reduce turning time.

This theoretical, point-mass evaluation addresses the direct force effects of thrust vectoring on the rotation of the aircraft velocity vector through the turn. This represents only a first approximation to "agility metrics" for close-in combat. Body-axis pitch and yaw rates (which include components of velocity vector rotation) as well as time-to-bank are of prime importance in this combat arena for pointing guns or short-range missiles at a target. The point-mass approach also ignores the issues associated with the moments generated by vectoring the thrust. As pointed out, vectoring thrust, particularly through large angles, can both generate significant moments on the aircraft (when jet exhaust is vectored at the tail) and induce jet effects (suckdown), depending on the nozzle location. Counterbalancing aerodynamic moments and/or the jet-induced flowfield may nearly negate the direct force effects of thrust vectoring, yielding little improvement in turn capability (velocity vector and flight-path angle control). However, aircraft pitch and/or yaw response may be significantly affected so as to enhance pointing response for close-in combat. Although coordinated aircraft response may be of greater consequence than pure "velocity rotation" for real-world aircraft and combat, these highly configuration-dependent issues were beyond our scope and intent to examine thrust-vectoring effects on gross maneuverability and compare results with other methods of reducing turning time.

Maneuver

The maneuver is defined by specifying initial and final conditions. The turn is initiated with the aircraft in straight (zero heading angle) and level (zero flight-path angle) flight at an altitude of 13,990 ft (4264 m) and a specified initial velocity. The maneuver is completed when the aircraft's heading angle reaches 180 deg with zero flight-path angle.

Equations of Motion

The equations of motion for flight of a point-mass aircraft over a flat earth are derived in Ref. 11. Ignoring sideforce, rearranging terms leads to

$$\dot{X} = V \cos\gamma \cos\chi \tag{1}$$

$$\dot{Y} = V \cos\gamma \sin\chi \tag{2}$$

$$\dot{h} = V \sin\gamma \tag{3}$$

$$\dot{V} = g\ [(T/W) \cos\epsilon \cos\nu - (D/W) - \sin\gamma] \tag{4}$$

$$\dot{\chi} = (g/V\cos\gamma)\ [(T/W)(\cos\epsilon \sin\nu \cos\mu + \sin\epsilon \sin\mu) + (L/W)\sin\mu] \tag{5}$$

$$\dot{\gamma} = (g/V)\ [(T/W)(\sin\epsilon \cos\mu - \cos\epsilon \sin\nu \sin\mu) + (L/W)\cos\mu - \cos\gamma] \tag{6}$$

These equations are written in the wind axes and describe the aircraft motion with respect to an Earth-fixed coordinate frame. The state variables are X, Y, h, V, χ, and γ. The variables μ, ϵ, and ν are specified control parameters. The aircraft weight W and the gravitational acceleration g are assumed constant during the maneuver. The initial altitude for all turns is 13,990 ft (4264 m). The gravitational acceleration at that altitude, $g = 32.131$ ft/s^2 (9.794 m/s^2),[10] is used as the constant value during the turn. The forces L, D, and T will be discussed next and will give rise to two more control variables.

Aircraft Characteristics

From the common coefficient forms of the aerodynamic lift and drag and the assumptions of a linear lift-curve slope (up to stall) and a parabolic drag polar, two expressions are derived.

$$L/W = (\rho V^2 S_W / 2W)\ C_{L_\alpha}\alpha \tag{7}$$

$$D/W = (\rho V^2 S_W / 2W)\ [C_{D_0} + K_1 (C_{L_\alpha}\alpha)^2] \tag{8}$$

Since the maximum available thrust remains constant during the turn, thrust can be expressed in terms of a control variable π, referred to as the throttle or power setting, and the thrust-to-weight ratio becomes.

$$T/W = (T/W)_{\max}\pi \tag{9}$$

The formulation of lift, drag, and thrust-to-weight ratios has introduced two control variables, α and π, and several aircraft parameters. The values of these parameters complete the specification of the aircraft model and are

$W = 12{,}150$ lb (5510 Kg)	$S_W = 237$ ft^2 (22 m^2)
$K_1 = 0.05$	$C_{D_0} = 0.02$
$C_{L_\alpha} = 5.0$ (per rad)	$C_{L_{\max}} = 1.0$
$(L/W)_{\max} = 7.22$	$(T/W)_{\max} = 1.5$

As pointed out by Brinson,[8] the induced drag is unrealistically low. Also, the assumption of a parabolic drag polar will be quite optimistic near stall, since significant flow-separation drag will generally be encountered. The thrust-to-weight ratio of 1.5 is considerably higher than that normally achieved by modern high-performance aircraft. However, these values, as optimistic or dissimilar to real-world aircraft as they may be, were necessarily chosen to agree with previous studies so results may be compared meaningfully. The values of $(T/W)_{max}$ and K_1 were varied to examine the effects of direct sideforce and thrust vectoring (Refs. 8 and 9, respectively) on turning times for more realistic aircraft models.

Control Variable Constraints

Two control variables, the throttle setting and the angle of attack, are constrained by physical considerations. The minimum thrust was taken to be 0; thus, the throttle setting must be between 0 and 1. The angle of attack is limited by both the maximum lift limit $C_{L_{max}}$ and the maximum load factor L/W_{max}. The effect on angle of attack is shown as a function of airspeed in Fig. 1. The velocity at which these two limits meet is the corner velocity V_C. The corner velocity is the velocity at which the aircraft achieves its maximum attained (instantaneous) turn rate and therefore plays a very important role in minimum-turning-time problems.

No constraints were placed on the thrust angle of attack and thrust sideslip angle. Although it does not appear that this is physically practical,[2,3] these angles were allowed full range in order to determine how much range of thrust vectoring would be exploited if it were available. The effects of constraining the two thrust angles were briefly examined and discussed in Ref. 9. No constraint was placed on the bank angle.

Optimal Control Problem

This formulation of the minimum-turning-time problem involves first-order nonlinear differential equations and partial specification of initial and final conditions on the state variables. The optimal control problem is to determine, out of all possible programs for the control variables, the one program that minimizes or maximizes a terminal quantity while simultaneously satisfying the required state-variable initial and final conditions. This is a two-point boundary-value problem that unfortunately cannot be solved in closed form. A numerical solution is required.

Many methods are available to solve this type of problem and are widely reported in the literature. A steepest-ascent method, presented in detail by Bryson and Denham,[12] was chosen to determine the optimal controls because it allows incorporation of state- and control-variable inequality constraints.[13,14] The procedure begins with a nonoptimal control-variable program. The equations of motion are integrated forward from the initial conditions using these nominal controls. In general, the resulting state-variable time histories will not satisfy the specified final conditions. Small perturbations of the control variables about the nominal trajectory are considered to drive the terminal quantities to their specified values while simultaneously extremizing a pay-off function. For this problem, the pay-off function is the final time. By continuing this process along the direction of steepest ascent (or descent) in the control-variable hyperspace, a control-variable program that minimizes the time to turn while satisfying final conditions is obtained.

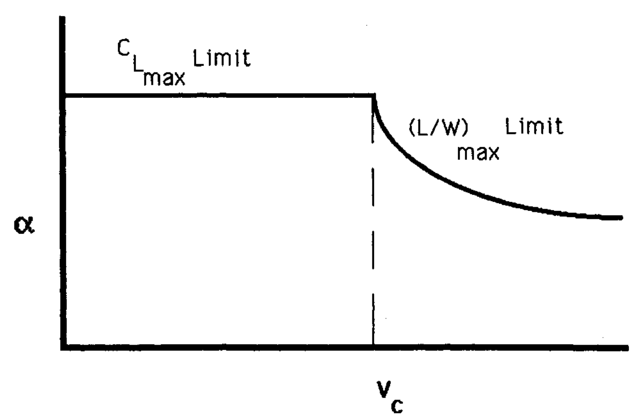

Fig. 1 Maximum angle of attack vs velocity.

It should be noted that neither the steepest-ascent method nor any other method for numerically solving this type of optimal control problem is guaranteed to find globally optimal solutions. Only relative minima or maxima may be found. The determination of a global extremum must be made by examining all of the local extrema, if possible. Since all of the techniques considered have this same drawback, it was not a factor in the choice of which method to use.

Results were obtained by running a FORTRAN 5 computer program on a CDC CYBER 845 computer. Total program length was 7400 words (including code, storage for local variables, arrays, constants, temporaries, etc., but excluding buffers and common blocks). Execution times were on the order of one central processor second per iteration. The number of iterations required to converge to a solution was highly dependent on the initial control-variable program, the choice of which was in itself an iterative process based on whether or not convergence seemed likely.

Results

Three cases with the nominal aircraft described earlier are presented here to show the benefits of thrust vectoring in reducing turning time. The three cases, each at a different initial velocity, were chosen so comparisons against other methods of reducing turning time could be made. The best, or most optimal, results are given in Table 1 and will be discussed in detail. Table 2 is a summary of turning times obtained for the various methods of reducing turning times discussed earlier. Times obtained by using vectored thrust are up to 1-1.5 s faster than with any other method of maneuver enhancement and up to 2.5 s faster than those for a standard aircraft (no maneuver enhancement capability, three-dimensional turns[4]).

Turning time is not the only measure of agility for high-performance aircraft. In particular, specific energy, given by

$$E = h + (V^2/2g) \tag{10}$$

is an important practical consideration in air-to-air combat maneuvering. As high an energy level as possible is desired so that kinetic and potential energies may be traded to one's advantage. A maneuver that ends up lower and slower than an adversary may negate the advantage of having turned faster. Energy gains/losses through the maneuvers are shown in Table 3. No attempt was made here or in the other referenced studies [4-6,8] to optimize turning performance with respect to

Table 1 Turning times with thrust vectoring

V_i, ft/s (m/s)	V_f, ft/s (m/s)	h_f, ft (m)	Time, s
420 (128)	662 (202)	11,866 (3617)	10.21
621 (189)	690 (210)	13,219 (4029)	8.24
903 (275)	660 (201)	13,714 (4180)	8.60

Table 2 Turning times for various methods[a]

	Turning times, s				
V_i, ft/s (m/s)	Vectored thrust	Three-dimensional turn[4]	Thrust reversal[5]	Vertical plane[6]	Side force[8]
420 (128)	10.21	—	—	—	10.36
621 (189)	8.24	10.5	9.55	9.27	9.47
903 (275)	8.60	11.2	10.25	10.17	10.68

[a]Initial altitude for three-dimensional turn was 13,390 ft (4081 m). All other initial altitudes were 13,990 ft (4264 m).

Table 3 Specific energy gains/losses for various methods[a]

V_i, ft/s (m/s)	Vectored thrust	Three-dimensional turn[4]	Thrust reversal[5]	Vertical plane[6]	Side force[8]
	Energy change, ft (m)				
420 (128)	1951 (595)	—	—	—	6711 (2046)
621 (189)	637 (194)	2719 (829)	5553 (1693)	−3484 (−1062)	5530 (1686)
903 (275)	−6186 (−1885)	3771 (1149)	−6405 (−1952)	1156 (352)	−4473 (−1363)

[a] g was taken to be 32.131 ft/s² (9.794 m/s²), the value at the initial altitude of 13,990 ft (4264 m).

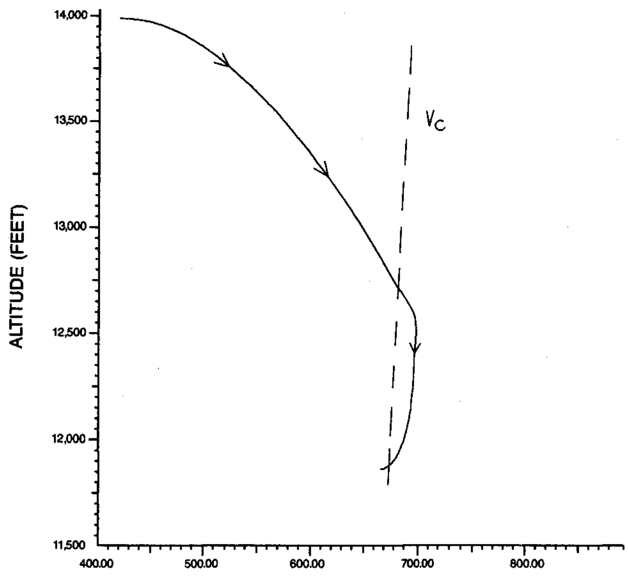

Fig. 2 Altitude vs velocity for V_i = 420 ft/s (128 m/s) (time to turn 10.21 s).

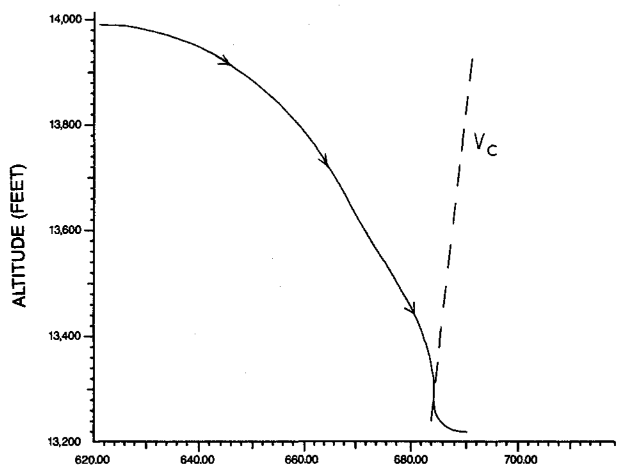

Fig. 3 Altitude vs velocity for V_i = 621 ft/s (189 m/s) (time to turn 8.24 s).

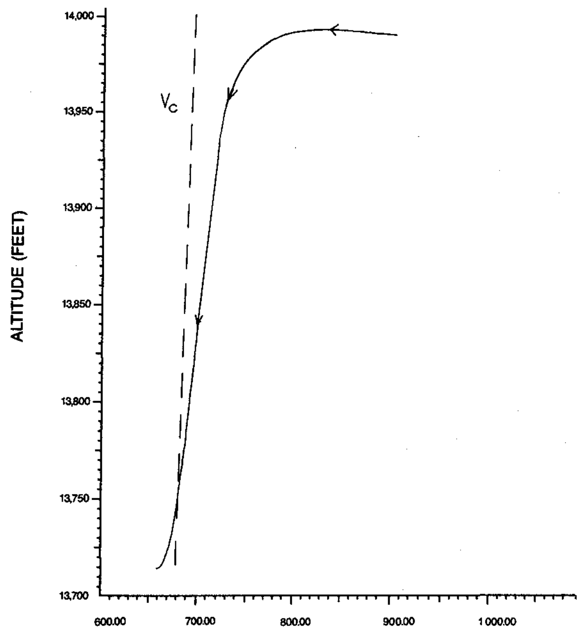

Fig. 4 Altitude vs velocity for V_i = 903 ft/s (275 m/s) (time to turn 8.60 s).

specific energy. We can only point out the general (and not surprising) observation that faster turning times were achieved at the expense of specific energy.

Corner Velocity

The corner velocity V_c is 962 ft/s (211 m/s) at the initial altitude of 13,990 ft (4264 m) and varies very little over the altitude ranges encountered, i.e., from $V_c = 700$ ft/s (213 m/s) at 14,700 ft (4481 m) to $V_c = 670$ ft/s (204 m/s) at 12,000 ft (3658 m). The importance of the corner velocity is that it is the velocity at which the aircraft achieves its maximum turn rate. It was expected and found to be true that trajectories that stay closer to the corner velocity result in faster turning times. Figures 2–4 show how the aircraft flew to and maintained V_c for the low, medium, and high initial airspeed cases, respectively.

Thrust Vectoring

The greater the aircraft's initial velocity, the more thrust-vectoring capability was used to get the aircraft to, and keep it at, its corner velocity. As shown in Fig. 5, for a low initial velocity [$V_i = 420$ ft/s (128 m/s)], only slightly over 12 deg of thrust angle of attack and 3 deg of thrust sideslip are used. The reduction on the thrust angles to zero corresponds to the throttle being "chopped" from nearly full power to zero partway through the maneuver.

For a higher initial velocity [$V_i = 621$ ft/s (189 m/s)], Fig. 6 shows the ranges of thrust vectoring increase to 70-deg angle of attack and 8-deg sideslip as partial or full throttle is used throughout the turn. At the highest initial velocity considered [$V_i = 903$ ft/s (275 m/s)], Fig. 7 shows an even greater increase in the range of thrust angles used, to 90-deg angle of attack and 180-deg sideslip (thrust reversal).

Thrust vectoring cannot increase the aircraft's available thrust, nor can it increase the aircraft's velocity. Therefore, this capability had very little effect on the low initial airspeed case where additional velocity was needed to significantly reduce the turning time. Consequently, only a minor improvement in turning time was found in the low initial airspeed case.

The benefit of vectored thrust is apparent at the two higher initial velocities. Thrust is vectored through large angles and turning times are reduced significantly over those of previous studies (Refs. 4–6, and 8). The most effective use of thrust vectoring is seen at the highest initial airspeed, $V_i = 903$ ft/s (275 m/s). The maneuver was flown at full throttle with the thrust initially vectored to 180 deg, effectively reversing thrust and quickly decelerating the aircraft to its corner velocity. The

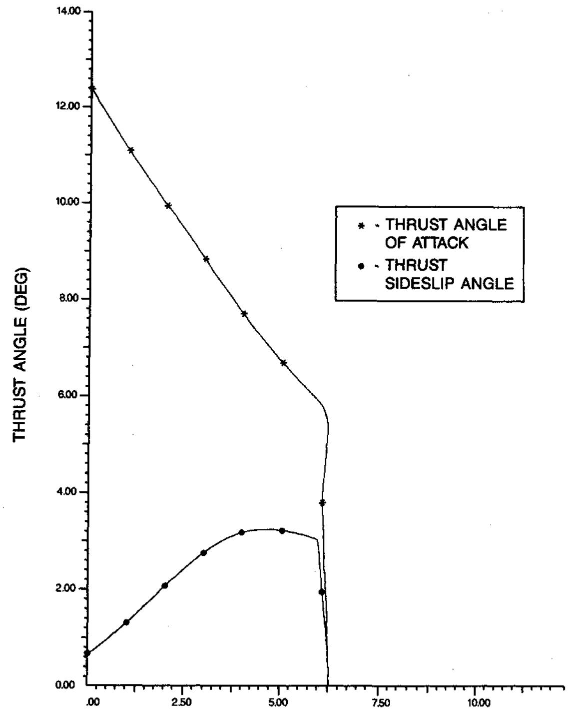

Fig. 5 Thrust angles vs time for V_i = 420 ft/s (128 m/s) (time to turn 10.21 s).

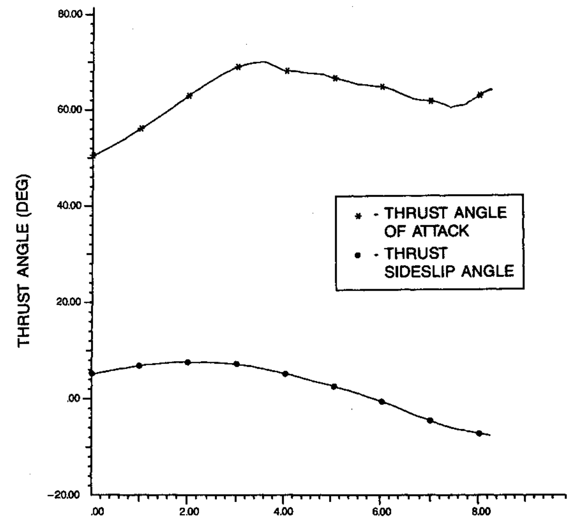

Fig. 6 Thrust angles vs time for V_i = 621 ft/s (189 m/s) (time to turn 8.24 s).

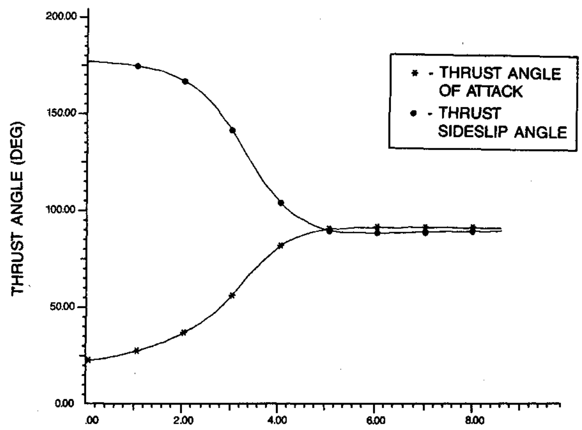

Fig. 7 Thrust angles vs time for V_i = 903 ft/s (275 m/s) (time to turn 8.60 s).

thrust angles were then modulated to keep the aircraft at V_c [approximately 690 ft/s (210 m/s)] for the rest of the turn.

In all three cases, the thrust was directed into the turn, supplementing the aircraft's lift. However, the low level of assumed (parabolic) drag allowed more than a realistic amount of thrust vectoring at elevated angles of attack (lift coefficient) for the actual velocity history.

Other Controls

The optimal program for the angle of attack is to maintain maximum angle of attack as dictated by the velocity in Fig. 1. The optimal solutions exhibited this characteristic. The bank-angle programs agree with the trends reported by Well and Berger.[7] For initial velocities less than the corner velocity, the aircraft banked and descended to accelerate toward V_c. For initial velocities greater than the corner velocity, the aircraft bank angle was much less as airspeed was bled off to decelerate to V_c. However, because of the thrust-reversal capability, the use of the vertical plane to gain or lose airspeed was not very prominent.

Conclusions

For an idealized case, major reductions in turning time can be realized through the use of vectored thrust. The higher the initial velocity, the greater the reduction in turning time. For an initial velocity of 903 ft/s (275 m/s) at an initial altitude of 13,990 ft (4264 m), the use of vectored thrust reduces the time to turn by 2.5 s, representing about 20% improvement. Thrust vectoring was used to supplement the aircraft's lift by directing the thrust into the turn. The results verify the intuitive approximation of adding thrust-vector effects directly to the lift-axis force component to calculate an average velocity during the turn and the corresponding turning times.

The steepest-ascent method, as implemented in this study, is heavily influenced by the choice of initial control-variable programs. More optimal solutions may be obtained with different starting control-variable programs. Although the results obtained in this study may not reflect the most optimal uses of thrust vectoring, even these less-than-optimum solutions show the dramatic improvements in turning time realizable with vectored thrust. Any more optimal solutions would only further advance this point. However, it must be noted that the unrestricted thrust-vectoring freedom assumed in pitch and sideslip will tend to yield optimistic results, as will the assumption of instantaneous control inputs and outputs.

In future investigations of thrust vectoring to reduce turning time, the problem may be simplified by keeping constant full throttle and maximum angle of attack (as dictated by the velocity, Fig. 1). This would reduce the number of controls to three and allow solutions to be displayed in the three-dimensional control space.

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